

# GLLH EM Invisible Cloak With Novel Front Branching And Without Exceed Light Speed Violation

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In this paper, we propose new Global and Local (GL) electromagnetic (EM) cloaks with distinctive class material  $a_{\alpha\beta} \log^\alpha(b_{\alpha\beta}/h)h^\beta$  (GLLH Cloak) without exceed light speed violation. The refractive index of the GLLH cloak material,  $n(r)$ , is large than one or equal to one. Our GLLH cloak is created by GL EM modeling and GL EM cloak inversion with searching class  $a_{\alpha\beta} \log^\alpha(b_{\alpha\beta}/h)h^\beta$ . The GLLH cloaks in this paper have finite speed and have no exceed light speed? physical violations and have more advantages. The GLLH EM cloaks can be practicable by using normal materials and are available for all broad frequency band. The GL EM cloak inversion and electromagnetic integral equation for cloak are presented in this paper. The novel EM wave propagation and front branching in the GLLH cloak by GL EM modeling are presented in this paper. The EM wave front propagation in GLLH cloak is behind of the front in free space. At time steps  $118dt$ , in the GLLH cloak, the wave front is curved as a crescent like and propagates slower than the light in free space. At the time step  $119dt$ , the EM wave inside of the GLLH cloak propagates slower than light speed, moreover, its two crescent front peaks intersect at a front branching point. At the front branching point, the front is split to two fronts. The novel front branching and crescent like wave propagation are displayed in the following figure 1, figure 2 and figures 5 -20 in this paper. All copyright and patent of the GLLH EM cloaks and GL modeling and inversion methods are reserved by authors in GL Geophysical Laboratory.

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## I. INTRODUCTION

In this paper, we propose new Global and Local (GL) electromagnetic (EM) cloaks with distinctive class material  $a_{\alpha\beta} \log^\alpha(b_{\alpha\beta}/h)h^\beta$  (GLLH Cloak) with finite speed, and without exceed light speed violation. The refractive index of the GLLH cloak material,  $n(r)$ , is larger than one or equal to one. Where  $a_{\alpha\beta}$  is coefficients,  $h = r - R_i$ . Our GLLH cloak is created by GL EM modeling and GL EM cloak inversion with searching class  $a_{\alpha\beta} \log^\alpha(b_{\alpha\beta}/h)h^\beta$ . We name our cloak as GLLH cloak. The GLLH EM cloaks can be practicable by using normal materials and are available for all broad frequency band. In 2001, we used Global Integral and Local Differential (GILD) EM modeling and inversion to detect the fly model imaging from EM sources in some frequency band. A strange double layer clothes phenomenon was discovered in the residual field. The double layer cloth protects the fly object from the exterior EM field detection. We called the strange phenomena as GILD effect and published it in paper [1]. In this paper, We present the double layer clothes imaging again in Figure 3 and Figure 4. For investigating the strange double layer clothes phenomena, we developed Global and Local field electromagnetic modeling method [2] and GL Metro Carlo inversion method [3]. These new methods

are complete different from the traditional Finite difference scheme [4] and Finite Element Method [5]. The GL method has totally advantages than the traditional methods. The big matrix solving cost and error reflection on the artificial boundary are difficulties in the traditional numerical simulation methods. The GILD EM modeling and inversion [6] are important progress to overcome these difficulties. The GL EM modeling and GL Metro Carlo inversion have advantages to completely overcome these difficulties in history [2] and [3]. In the physical and quantum mechanical simulations, Born approximation [7] is often used. However, the Born approximation is not accurate and can not be used in the high frequency band and high contrast and singularity materials. We used the GL EM modeling and inversion to simulate the double layer clothes phenomena [1] and discover a mirage image [8], and proposed GL double layer cloak [9-13]. Pendry at al proposed the single layer cloak by transform optical technology in [14] (Ps cloak). However, the *exceed light speed* and *infinite EM field speed* are its two main physical violations. In [15], we proved a theorem that there exists no Maxwell wavefield can be excited by sources inside the single layer cloaked concealment, if the cloak material parameters are finite, and the concealment is fill by basic EM materials  $\epsilon_0$  and  $\mu_0$ . By the theorem, there is no EM wave can be excited in Ps' cloaked concealment that is its thrird physical violation. Our GL double layer cloak [9-13] overcame this difficulty. The GL outer layer cloak has invisibility functions. The GL inner layer cloak decays the internal wave field, such that the internal wave field can not propagate

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to outside of the inner layer. There is no exceed light speed violation in the GL inner layer cloak. In the GL outer layer cloak, we obtained reduced degenerate cloak material, when  $r \rightarrow R_1$   $\varepsilon_r \mu_\theta \approx (r - R_1) \rightarrow 0$ . However, Ps cloak is strong degenerate material, in which[14] the  $\varepsilon_r \mu_\theta = R_2^2(r - R_1)^2/r^2/(R_2 - R_1)^2$ , when  $r \rightarrow R_1$   $\varepsilon_r \mu_\theta \approx (r - R_2)^2 \rightarrow 0$ . We proposed a GLWF double layer EM cloaks in broad frequency band [16] to overcome the exceed light speed. The Figure 9 in [16] shows that the EM wave propagation in Ps cloak does exceed light speed. The Figure 7 in [16] exhibits that the EM wave propagation in GLWF double layer cloak does not exceed light speed. As a great jump of the GLWF cloak [16], by using GL Metro Carlo inversion [3], we propose the new GLLH cloaks with class material  $a_{\alpha\beta} \log^\alpha(b_{\alpha\beta}/h)h^\beta$ , one of which is formulized by (2)-(5) in this paper. The cloak has nonzero and positive  $\varepsilon_r$ ,  $\mu_r$ ,  $\varepsilon_\theta$ , and  $\mu_\theta$  in whole cloak domain  $R_i \leq r \leq R_o$ , that is its large advantage over than the 2rd order strong singular Ps cloak with zero parameters on the inner boundary,  $\varepsilon_r = 0$  and  $\mu_r = 0$ . We proved that the refractive index of our GLLH cloak is larger than one or equal to one,  $n(r) = \sqrt{\varepsilon_r \mu_\theta} \geq 1$ , in whole cloak domain  $R_i \leq r \leq R_o$ , when  $R_i = 0.249$ ,  $R_o = 0.47$ . By using GL EM modeling [2], the novel full EM wave propagation in the cloak show that there is no *exceed light speed* violation in our GLLH cloak in in this paper. The novel EM wave propagation and distinct front branching in the GLLH cloak by GL EM modeling are presented in in this paper. The EM wave front in GLLH cloak is behind of the front in free space. The figures 8-13, show that the wave front is successively curved as a crescent like and propagates slower

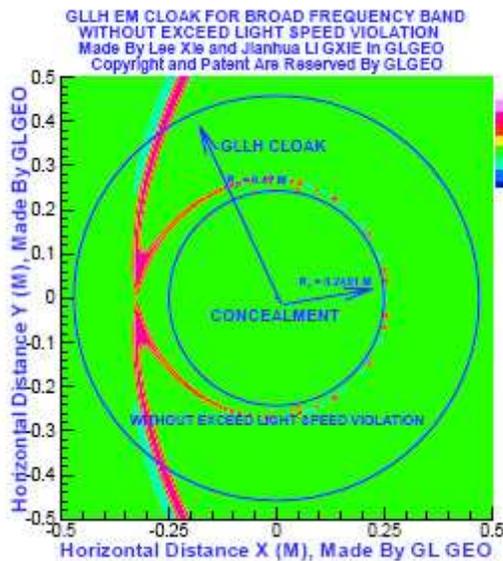


FIG. 1: (color online) Electric wave propagation at time step  $118dt$ , the wave front is curved as the crescent like, red S in right is source location

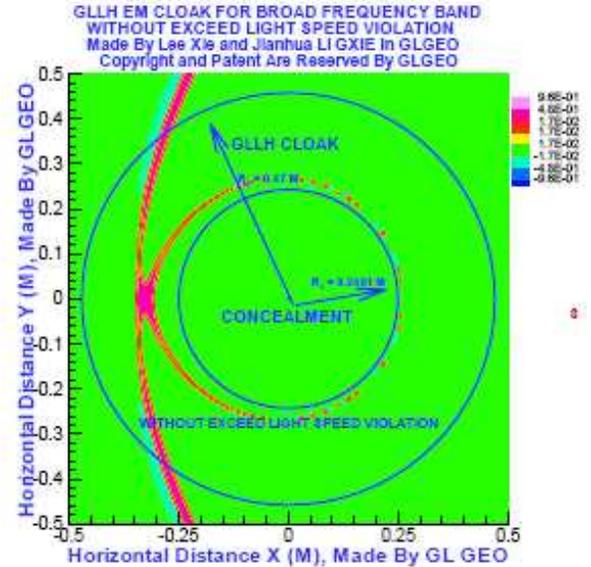


FIG. 2: (color online) Electric wave propagation at time step  $119dt$ , the two peaks of the crescent like wave front intersects at a branching point, red S in right is source location

than the light in free space. In particular, at time steps  $118dt$ , in the GLLH cloak, the figure 1 shows the curved EM wave front is much similar with the crescent. At the time step  $119dt$ , the electric wave inside of the GLLH cloak propagates slower than light speed, moreover, its two crescent peaks intersect at a novel front branching point. At the front branching point, the front is split to two fronts. One is outgoing front, which is propagating forward to out of the cloak and bring out most wave

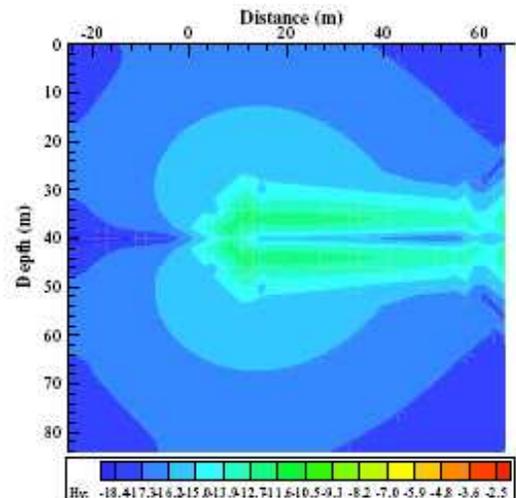


FIG. 3: (color online) A novel double layer cloak surround space shuttle model to prevent detection has been discovered in GILD EM inversion in 2001, which was published in SEG expand abstract in 2002[] .

energy. After the outgoing front propagating out of the GLLH cloak, it is recovered to the original wave front in free space. Other front is similar with a sphere surface wave and is attractive propagating to the inner boundary. The attracting front propagation is very slower. Its amplitude and speed are rapidly decay to zero. The attracting front is propagating to inner boundary, but it can not reach to the inner boundary  $r = R_i$ . The novel front branching and crescent like wave propagation are displayed in the figure 1 figure 2 and figure 5 -20 in this paper.

When source is located outside of the GLLH cloak, the observer point  $r$  is located inside of the cloak and it is going to the inner boundary  $r = R_i$ , the EM wave field delay to zero inverse radial. On the inner boundary, the radial electric wave field  $E_r(R_i, \theta, \phi) = 0$  and the radial magnetic wave field  $H_r(R_i, \theta, \phi) = 0$  are distinctive property of the GLLH invisible cloak. On the inner boundary of other transformed cloak, the radial wave EM field are not zero. There is a novel geometry for our GLLH invisible cloak.

Ulf Leonhardt et al. proposed a new cloak with finite speed based on a Euclid and non Euclid joint transform[17]. The ULF cloak overcomes the "infinite speed" physical violation, even though its refractive index less than one in some subdomain, for example,  $n(\sigma, \sigma', \tau) = n(0.75\pi, \pi, \tau) < 1$ . Chen et al and Zhang et al proposed an analysis for Ps cloak using Mie transform [24][25].

The creating motivation of our GLLH cloak is according to our double layer cloth observation in 2001 which is shown in Figure 3 and Figure 4, and our 3D FEM super convergence observation in 1973. [5][18][19].

The form  $a_{\alpha\beta} \log^\alpha(b_{\alpha\beta}/h)h^\beta$  is used as estimation form in number theory [20][21][22], PDE, numerical mathematics[5][23], probability, geometry, and in other estima-

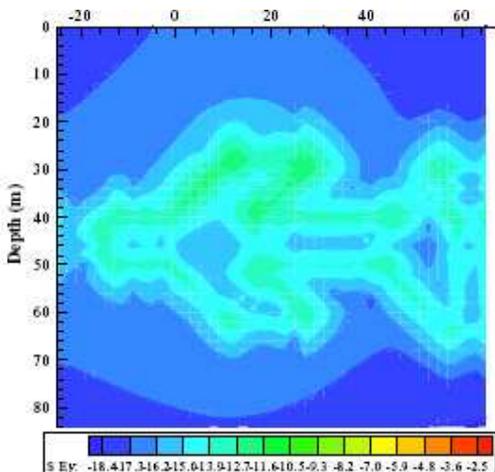


FIG. 4: (color online) A novel double layer cloak surround fly model anti detection has been discovered in GILD EM inversion in 2001, which was published in SEG expand abstract in 2002[] .

tion theory in physics and other science. However, no one think it can be used in invisible cloak material before this paper. Fortunately, the form  $a_{\alpha\beta} \log^\alpha(b_{\alpha\beta}/h)h^\beta$  is used as search class of the GL cloak inversion to create the GLLH invisible cloak that is our inspiration and discovering. The finding inversion is to find object from the observer measured scattering wave from the object and is excited by control or natural sources. The invisible cloak generation is hiding inversion. The invisible and hiding inversion is to generate a cloak material such that the scattering wave from the cloak is zero. The double layer cloth and fly imager in Figure 3 and Figure 4 show that the finding inversion and hiding inversion are contradiction and compromise each other.

The description order of this paper is as follows. Introduction is described in Section 1. In Section 2, we propose new GLLH em cloak without exceed light speed. The GL EM cloak inversion is proposed in Section 3. In Section 4, we present novel wave front branching and EM wave propagation in the GLLH cloak without exceed light speed. The distinct invisible properties of GLLH EM cloak are proposed in Section 5. In Section 6, we propose GLLH EM double layer cloak. In section 7, we present that the EM wave field decay to zero inverse radial. In section 8, we describe history and discussion. The conclusion of this paper is described in section 9

## II. GLLH EM CLOAK WITHOUT EXCEED LIGHT SPEED

In this section, we propose a new GLLH EM invisible cloak for broad frequency band and without exceed light speed violation.

### A. GLLH EM INVISIBLE CLOAK

In the Maxwell equation

$$\begin{aligned}\nabla \times \vec{E} &= -\bar{\mu} \frac{\partial \vec{H}}{\partial t} + \vec{M} \\ \nabla \times \vec{H} &= \bar{\epsilon} \frac{\partial \vec{E}}{\partial t} + \vec{J}.\end{aligned}\quad (1)$$

Where  $\vec{E}$  is electric wave vector field,  $\vec{H}$  is magnetic wave vector field

$$\bar{\epsilon} = \text{diag} [\epsilon_r \quad \epsilon_\theta \quad \epsilon_\phi] \epsilon_0, R_i \leq r \leq R_o, \quad (2)$$

$$\bar{\mu} = \text{diag} [\mu_r \quad \mu_\theta \quad \mu_\phi] \mu_0, R_i \leq r \leq R_o. \quad (3)$$

We propose the GLLH EM invisible cloak material parameters in 3D spherical coordinate system, the cloak material parameters are radial dependent as follows,

$$\begin{aligned}\epsilon_r = \mu_r &= \frac{1}{1+4R_o+2R_o^2} \left( \frac{2R_o^2(R_o-R_i)}{r^2} \right. \\ &+ \frac{2+6R_o}{\log(e^{1/R_o}(R_o-R_i)/(r-R_i))} \frac{(r-R_i)}{R_o r^2} \\ &\left. - \frac{1+2R_o}{\log^2(e^{1/R_o}(R_o-R_i)/(r-R_i))} \frac{(r-R_i)}{R_o^2 r^2} \right),\end{aligned}\quad (4)$$

$$\begin{aligned} \varepsilon_\theta = \varepsilon_\phi = \mu_\theta = \mu_\phi \\ = \frac{1}{2 \log(e^{1/R_o}(R_o - R_i)/(r - R_i))(r - R_i)} \\ + \frac{1}{2 \log^3(e^{1/R_o}(R_o - R_i)/(r - R_i))(r - R_i)} \end{aligned} \quad (5)$$

where the  $R_i \leq r \leq R_o$  is the spherical annular cloak domain,  $R_i$  is inner radius of the annular cloak,  $R_o$  is outer radius of the annular cloak,  $\bar{\varepsilon}$  is the dielectric parameter matrix,  $\bar{\mu}$  is the permeability parameter matrix,  $\varepsilon_0$  is the basic dielectric parameter,  $\mu_0$  is the basic magnetic permeability parameter,  $\varepsilon_r$  is relative dielectric in  $r$  direction,  $\varepsilon_\theta$  is relative dielectric in  $\theta$  direction,  $\varepsilon_\phi$  is relative dielectric in  $\phi$  direction,  $\mu_r$  is relative permeability in  $r$  direction,  $\mu_\theta$  is relative permeability in  $\theta$  direction,  $\mu_\phi$  is relative permeability in  $\phi$  direction.

## B. INVISIBLE FUNCTIONS OF GLLH EM CLOAK

### 1. function I

When the source  $r_s$  and observer  $r$  are located outside of the cloak, we verify the invisible function I of the GLLH EM cloak materials in (4)-(5) as follows:

*Exterior EM field must not be interfered by scattering from the cloak :*

$$\begin{bmatrix} E(r, r_s, t) \\ H(r, r_s, t) \end{bmatrix} = \begin{bmatrix} E_b(r, r_s, t) \\ H_b(r, r_s, t) \end{bmatrix}, \quad (6)$$

### 2. function II

When the source  $r_s$  is located outside of the cloak and observer  $r$  is inside of the cloak, we verify the invisible function II of the GLLH EM cloak materials in (4)-(5) as follows:

*Exterior EM field does not penetrate into concealment :*

$$\begin{bmatrix} E(r, r_s, t) \\ H(r, r_s, t) \end{bmatrix} = 0, \quad (7)$$

Our GLLH EM cloak completely overcomes the exceed light speed?and infinite speed? two physical violations. The GLLH EM cloak as GL outer layer cloak and the GL inner layer cloak (3)-(5) can be coupled to construct new GLLH EM double layer cloak.

## III. GL EM CLOAK INVERSION

We proposal a GL EM inversion for cloak in this section. The 3D EM integral equation and GL EM modeling in [2] and GL Metro Carlo inversion in [3] are base of the GL EM cloak inversion method.

## A. 3D EM INTEGRAL EQUATION

A 3D electromagnetic integral equation is proposed in this section as follow:

$$\begin{bmatrix} E(r, r_s, t) \\ H(r, r_s, t) \end{bmatrix} = \begin{bmatrix} E_b(r, r_s, t) \\ H_b(r, r_s, t) \end{bmatrix} + \int_{\Omega_{cloak}} G_{E,H,b}^{J,M}(r', r, t) *_t \begin{bmatrix} \delta D_{11} \\ \delta D_{22} \end{bmatrix} \begin{bmatrix} E(r', r_s, t) \\ H(r', r_s, t) \end{bmatrix}, \quad (8)$$

$$\begin{bmatrix} E(r, r_s, t) \\ H(r, r_s, t) \end{bmatrix} = \begin{bmatrix} E_b(r, r_s, t) \\ H_b(r, r_s, t) \end{bmatrix} + \int_{\Omega_{cloak}} G_{E,H}^{J,M}(r', r, t) *_t \begin{bmatrix} \delta D_{11} \\ \delta D_{22} \end{bmatrix} \begin{bmatrix} E_b(r', r_s, t) \\ H_b(r', r_s, t) \end{bmatrix}, \quad (9)$$

Where  $E(r, r_s, t)$  is the electric intensity field,  $H(r, r_s, t)$  the is magnetic intensity field,  $r$ , is observe space location variable,  $r_s$  is source space location variable,  $t$  is time,  $E_b(r, r_s, t)$  is the background electric intensity field,  $H_b(r, r_s, t)$  is the background magnetic intensity field,  $G_{E,H,b}^{J,M}(r', r, t)$  is  $6 \times 6$  EM Greens tensor matrix,  $diagonal[\delta D_{11}, \delta D_{22}]$  is  $6 \times 6$  variation EM material matrix as follows:

$$\begin{aligned} \delta D_{11} &= (\bar{\varepsilon} - \varepsilon_0 I) \frac{\partial}{\partial t}, \quad \delta D_{22} = (\bar{\mu} - \mu_0 I) \frac{\partial}{\partial t} \\ \bar{\varepsilon} &= diag[\varepsilon_r \quad \varepsilon_\theta \quad \varepsilon_\phi] \varepsilon_0, \\ \bar{\mu} &= diag[\mu_r \quad \mu_\theta \quad \mu_\phi] \mu_0, \end{aligned} \quad (10)$$

Where  $\bar{\varepsilon}$  is the dielectric parameter matrix,  $\bar{\mu}$  is the permeability parameter matrix, the dielectric and permeability can be isotropic or anisotropic materials,  $\varepsilon_0$  is the basic dielectric parameter,  $\mu_0$  is the basic magnetic permeability parameter,  $\varepsilon_r$  is relative dielectric in  $r$  direction,  $\varepsilon_\theta$  is relative dielectric in  $\theta$  direction,  $\varepsilon_\phi$  is relative dielectric in  $\phi$  direction,  $\mu_r$  is relative permeability in  $r$  direction,  $\mu_\theta$  is relative permeability in  $\theta$  direction,  $\mu_\phi$  is relative permeability in  $\phi$  direction, for the cloak material, the  $\bar{\varepsilon}$  and  $\bar{\mu}$  are proposed in formulas (3) – (5) in this paper.

## B. FUNCTIONS IN GL EM CLOAK INVERSION

We propose a GL EM cloak inversion for invisible cloak in this section. We request two invisible functions and without exceed light speed violation for the GL EM cloak: (I) the exterior EM field should not be scattering interfered by the cloak; (II) the exterior EM field can not penetrate into the concealment of the cloak; and (III) without exceed light speed violation.

## C. GL EM CLOAK EXTERIOR INVERSION

When the source  $r_s$  and observer  $r$  are located outside of the cloak, we present the cloak invisible function I as

the following inversion.

*Exterior EM field must not be interfered by scattering from the cloak.*

According to the invisible function I (6), and using the EM integral equation (8), we have

$$\int_{\Omega_{cloak}} G_{E,H,b}^{J,M}(r', r, t) *_t \begin{bmatrix} \delta D_{11} & \\ & \delta D_{22} \end{bmatrix} \begin{bmatrix} E(r', r_s, t) \\ H(r', r_s, t) \end{bmatrix} = 0, \quad (11)$$

Using the EM integral equation (9), we have

$$\int_{\Omega_{cloak}} G_{E,H}^{J,M}(r', r, t) *_t \begin{bmatrix} \delta D_{11} & \\ & \delta D_{22} \end{bmatrix} \begin{bmatrix} E_b(r', r_s, t) \\ H_b(r', r_s, t) \end{bmatrix} = 0, \quad (12)$$

#### D. GL EM CLOAK INNER INVERSION

When the source  $r_s$  is located outside of the cloak and observer  $r$  is inside of the cloak, we present the cloak invisible function II in the following inversion.

*Exterior EM field does not penetrate into concealment.*

According to the invisible function II (7) and using the EM integral equation (8), we have

$$\begin{bmatrix} E_b(r, r_s, t) \\ H_b(r, r_s, t) \end{bmatrix} + \int_{\Omega_{cloak}} G_{E,H,b}^{J,M}(r', r, t) *_t \begin{bmatrix} \delta D_{11} & \\ & \delta D_{22} \end{bmatrix} \begin{bmatrix} E(r', r_s, t) \\ H(r', r_s, t) \end{bmatrix} = 0, \quad (13)$$

By using the EM integral equation (9), we have

$$\begin{bmatrix} E_b(r, r_s, t) \\ H_b(r, r_s, t) \end{bmatrix} + \int_{\Omega_{cloak}} G_{E,H}^{J,M}(r', r, t) *_t \begin{bmatrix} \delta D_{11} & 0 \\ 0 & \delta D_{22} \end{bmatrix} \begin{bmatrix} E_b(r', r_s, t) \\ H_b(r', r_s, t) \end{bmatrix} = 0, \quad (14)$$

#### E. CONSTRAINT IN GLLH EM CLOAK INVERSION

We present the WITHOUT EXCEED LIGHT SPEED VIOLATION to be the constraint of the GLLH EM cloak inversion, for the radial dependent  $\bar{\varepsilon}(r)$  and  $\bar{\mu}(r)$ ,

$$\begin{aligned} \varepsilon_r(r)\mu_\theta(r) &\geq 1, \\ \varepsilon_\theta(r)\mu_r(r) &\geq 1, \\ \varepsilon_\theta(r) &= \varepsilon_\phi(r) = \mu_\theta(r) = \mu_\phi(r). \end{aligned} \quad (15)$$

#### F. GL EM CLOAK INVERSION

The EM integral equation (11) and (12) for  $r_s > R_o$ , and  $r > R_o$ , the EM integral equation (13) and (14) for  $r_s > R_o$ , and  $r < R_i$ , and the "No exceed light speed" radial dependent constraint (15) are coupled to construct the GL EM cloak inversion for the EM invisible cloak. We use the GL EM Metro Carlo inversion method in [3] to solve the GLLH EM cloak inversion, we find the radial dependent GLLH EM cloak material to satisfy the GL EM cloak exterior inversion (11) and(12); the GL EM cloak inner inversion (13) and (14); and constraint (15) without exceed light violation.

#### IV. NOVEL EM PROPAGATION IN GLLH EM CLOAK WITHOUT EXCEED LIGHT SPEED

In this section, we present a novel EM propagation in the GLLH EM cloak without exceed light speed. The EM wave propagation pattern completely is new, and never be shown in other authors' papers.

##### A. THE SIMULATION MODEL OF THE GLLH EM CLOAK

The simulation model: the 3D domain is  $[-0.5m, 0.5m] \times [-0.5m, 0.5m] \times [-0.5m, 0.5m]$ , the mesh number is  $201 \times 201 \times 201$ , the mesh size is 0.005m. The electric current point source is defined as

$$\delta(r - r_s)\delta(t)\vec{e}, \quad (16)$$

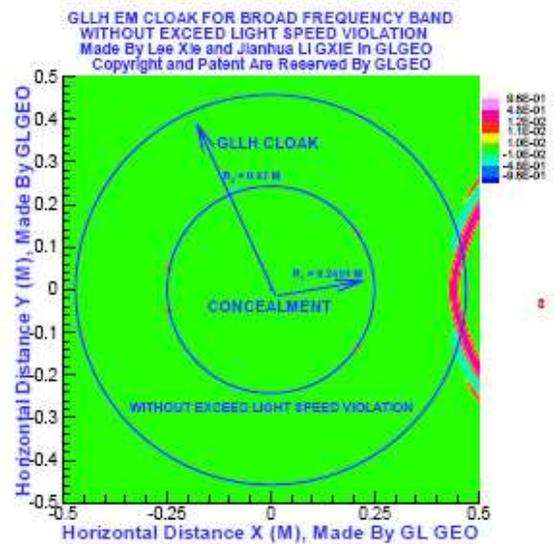


FIG. 5: (color online) At time step  $40dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates no faster than light speed. The red S in right is source.

where the  $r_s$  denotes the location of the point source, the unit vector  $\vec{e}$  is the polarization direction, the time step  $dt = 0.3333 \times 10^{-10}$  second, the frequency band is from 0.05 GHz to 15 GHz, the largest frequency  $f = 15\text{GHz}$ , the shortest wave length is  $0.02\text{m}$ . The GLLH EM cloak is consist of the spherical annular  $R_1 \leq r \leq R_2$  with center in original point and is fill in by the GLLH EM cloak materials , where  $R_1 = 0.2491\text{ meter}$ ,  $R_2 = 0.47\text{ meter}$ . The cloak is divided into  $90 \times 180 \times 90$  cells. The spherical coordinate is used in the sphere  $r \leq R_2$ , the Cartesian rectangular coordinate is used in outside the sphere  $r > R_2$  to mesh the domain.

## B. NOVEL ELECTRIC WAVE PROPAGATION

The electric intensity wave is excited by the point source  $S$ , which is denoted by red S in the figures in this paper. In Figures 1-2 and Figures 5-18, the source by red S is located in free space, in the right side outside of the whole GLLH EM cloak, at  $(0.83\text{m}, 0.0, 0.0)$ . In Figure 19, the point source by red S is located in the left outer side of the cloak, at  $(-0.83\text{m}, 0.0, 0.0)$ . In Figure 20, the point source by red S is located in the bottom outer side of the cloak, at  $(0.0, -0.83\text{m}, 0.0)$ .

In Figure 5, at time step  $40dt$ , *electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates no faster than light speed. Its front inside of GLLH EM cloak is behind of the electric wave front in free space

The Figure 6 shows that at time step  $50dt$ , *electric wave*,  $E_{xx}$  inside of the GLEM cloak  $R_1 \leq r \leq R_2$  propagates no faster than light speed.

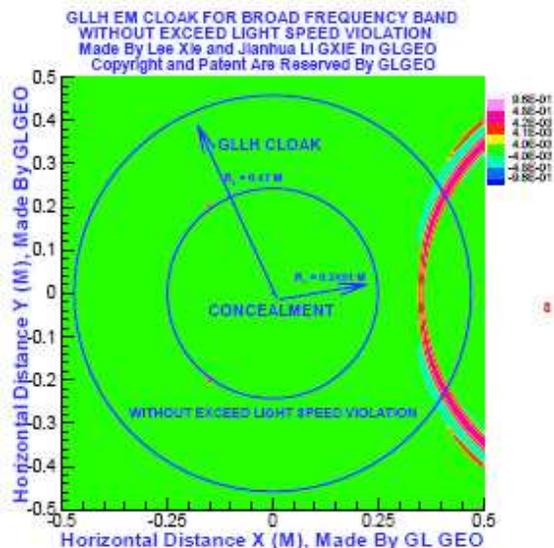


FIG. 6: (color online) At time step  $50dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates no faster than light speed. the red S in right is source.

In Figure 7, at time step  $60dt$ , *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. Its front inside of GLEM cloak is behind of the electric wave front in free space

The Figure 8 shows that at time step  $70dt$ , *electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed.

In Figure 9, at time step  $80dt$ , *electric wave*,  $E_{xx}$  inside of the GLEM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. Its front inside of GLLH EM cloak is behind of the electric wave front in free space

The Figure 10 shows that at time step  $90dt$ , *electric wave*,  $E_{xx}$  inside of the GLEM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed.

In Figure 11, at time step  $100dt$ , *electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. Its front inside of GLEM cloak is behind of the electric wave front in free space.

The Figure 12 shows that at time step  $110dt$ , *electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed.

In Figure 1 and Figure 13, at time step  $119dt$ , *electric wave*,  $E_{xx}$  inside of the GLEM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. Its front inside of GLLH EM cloak is behind of the electric wave front in free space.

The Figure 2 and Figure 14 shows that at time step  $120dt$ , *electric wave*,  $E_{xx}$  inside of the GLEM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. The upside and downside parts of curved electric wave front are intersected at a branching point. These branching points form a 2D subsurface which depends on the source loca-

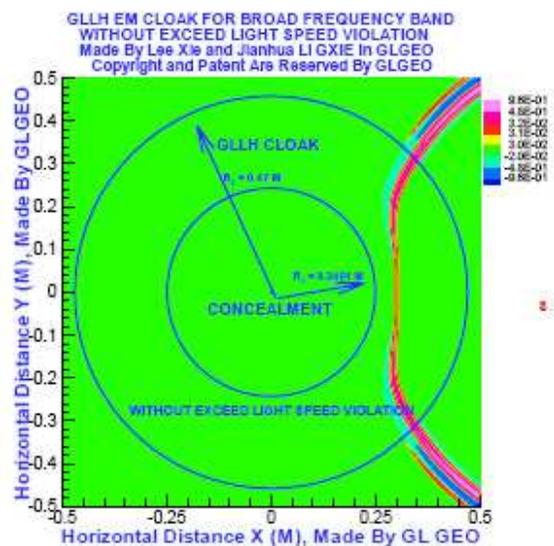


FIG. 7: (color online) At time step  $60dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. the red S in right is source.

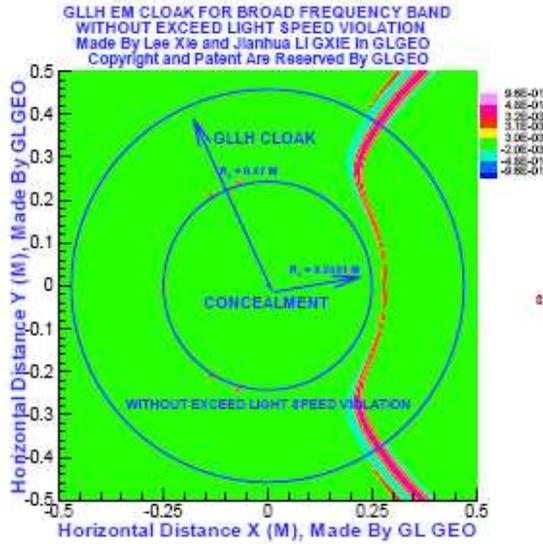


FIG. 8: (color online) At time step  $70dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed

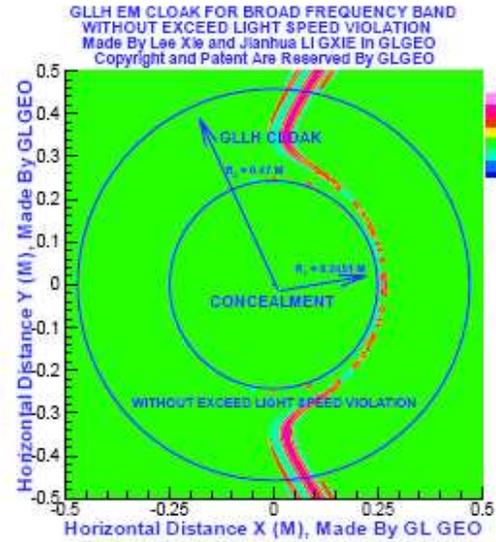


FIG. 10: (color online) At time step  $90dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed

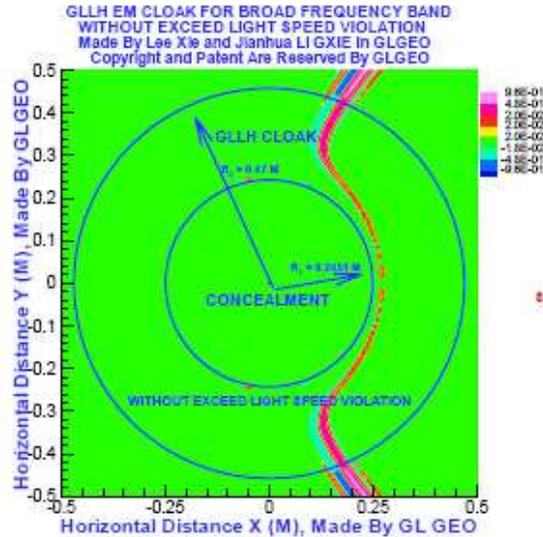


FIG. 9: (color online) At time step  $80dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed

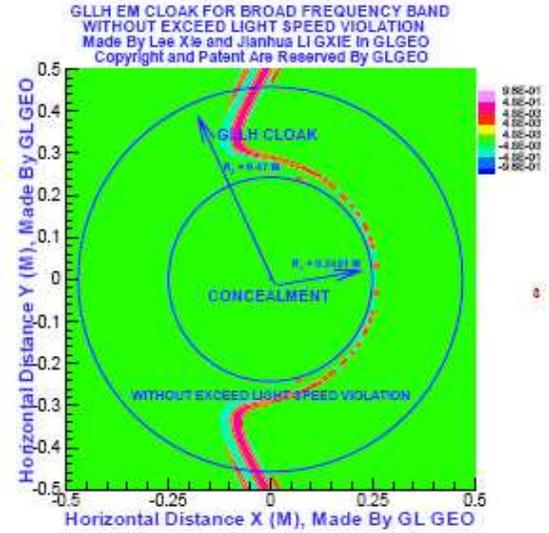


FIG. 11: (color online) At time step  $100dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed

tion. In this Figure and Figure 2, the red S denotes the source which is located in the right of the cloak. The front branching point is located in left of the concealment. In Figure 15, at time step  $121dt$ , *electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. At time step  $121dt$ , the wave front inside of GLEM cloak is split into two wave fronts at the branching point. One wave front continuously propagates outgoing and slower than the light speed. Other closed

wave front propagates attracting into the inner boundary  $r = R_1$  of the cloak and much slower than the light speed. The speed of the attracting wave front is going to zero. Its amplitude is rapidly decay to zero.

In Figure 16, at time step  $122dt$ , *electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. Its outgoing wave front inside of GLLH EM cloak is behind of the electric wave front in free space. Other attracting wave front propagates much

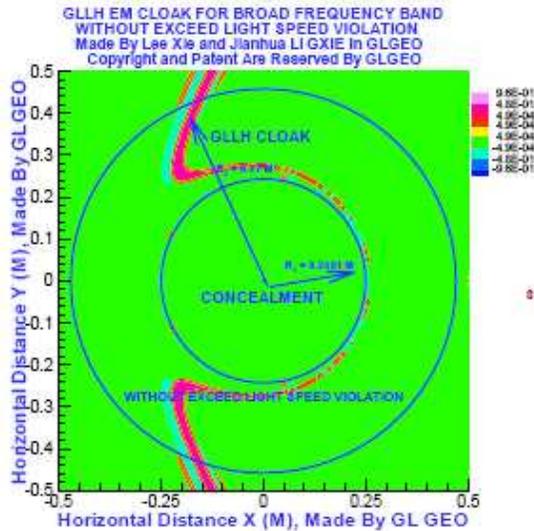


FIG. 12: (color online) At time step  $110dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed

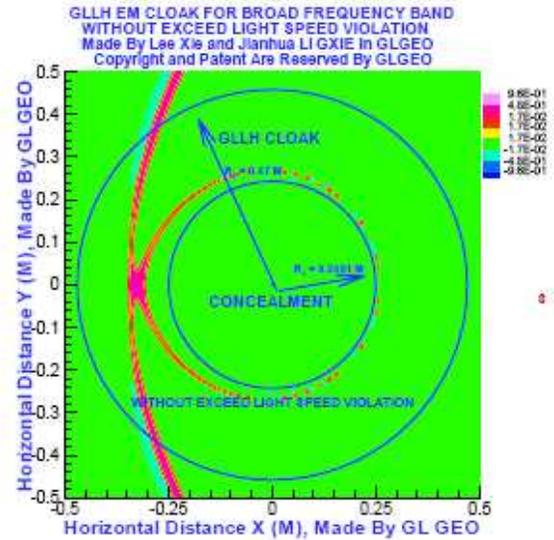


FIG. 14: (color online) At time step  $119dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. The curved front intersects at branching point in left which depends on the source. The red S in right is source.

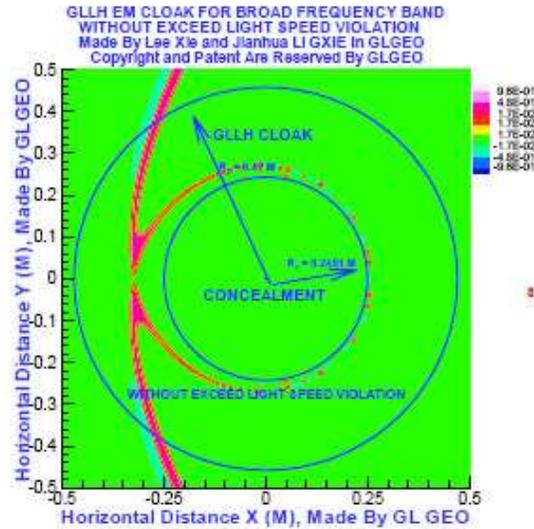


FIG. 13: (color online) At time step  $118dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed

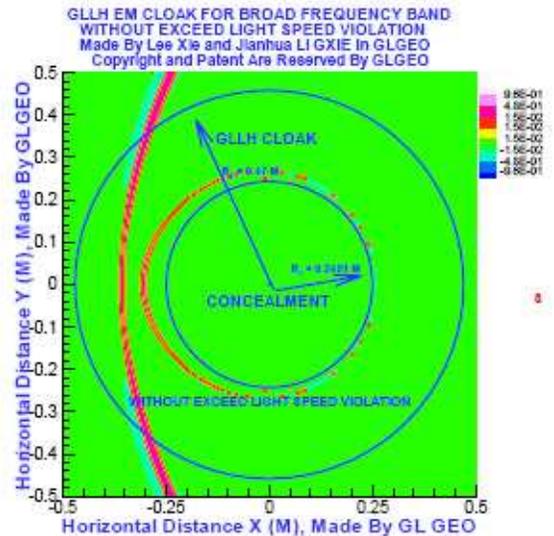


FIG. 15: (color online) At time step  $120dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. The curved front is split to two branching fronts

slow than the light speed. Its speed is going to zero. Its amplitude is rapidly decay to zero.

In Figure 17, at time step  $130dt$ , *electric wave*,  $E_{xx}$  inside of the GLEM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. Its outgoing wave front inside of GLLH EM cloak is behind of the electric wave front in free space. Other attracting wave front propagates much slow than the light speed. Its speed is going to zero.

In Figure 18, at time step  $133dt$ , the outgoing front of *electric wave*,  $E_{xx}$  propagates out of the GLEM cloak

$R_1 \leq r \leq R_2$  and recovers to wave in free space and is not scattering interfered by the cloak Other attracting wave front propagates much slow than the light speed. It propagates going to the inner boundary  $r = R_1$  of the cloak. Its speed is going to zero.

In the Figure 1 and Figure 2, and Figure 5-18, the

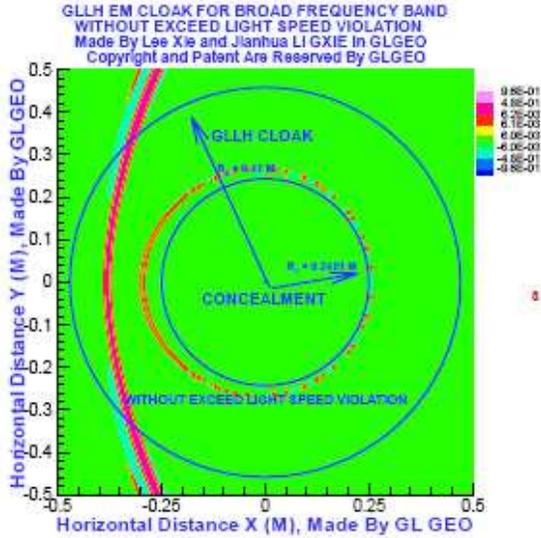


FIG. 16: (color online) At time step  $122dt$ , front of *Electric wave*,  $E_{xx}$  inside of the propagates slower than light speed

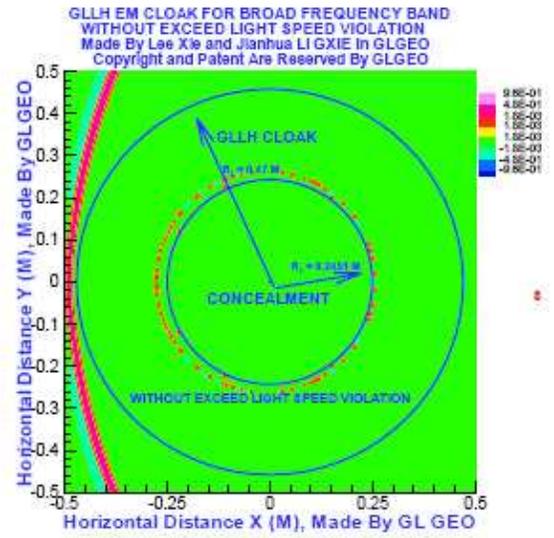


FIG. 18: (color online) At time step  $133dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed

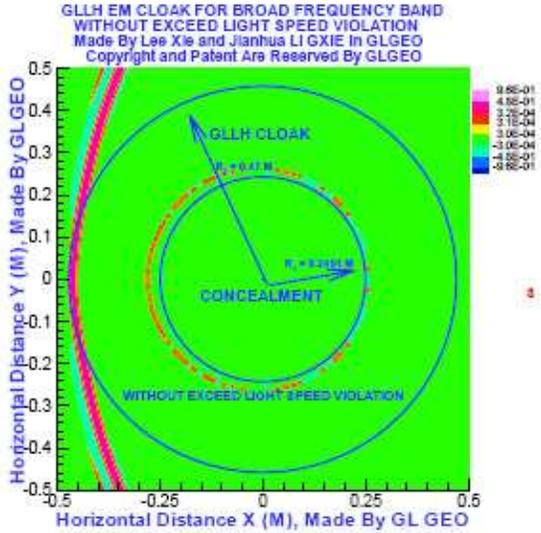


FIG. 17: (color online) At time step  $130dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed

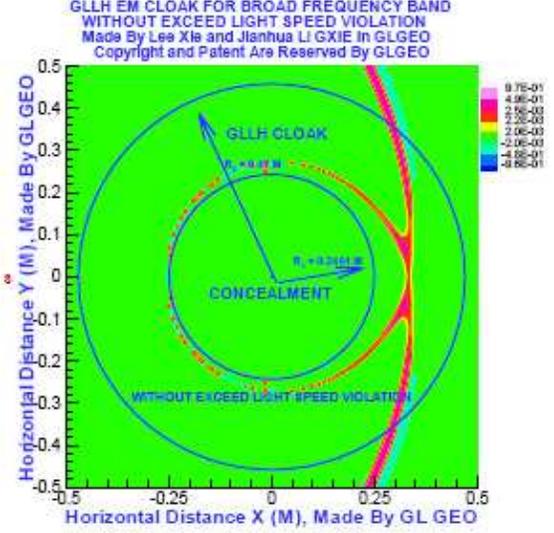


FIG. 19: (color online) At time step  $119dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. The red S in left is source.

source is denoted by the red S which is located in the right of the cloak.

Figure 19 shows that at time step  $120dt$ , *electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. The upside and downside parts of curved electric wave fronts are intersected at a branching point in right. These branching points form a 2D subsurface which depends on the source location. In this Figure, the red S denotes the source which is lo-

cated in the left of the cloak. The front branching point is located in right of the concealment.

In Figure 20, at time step  $120dt$ , *electric wave*,  $E_{xx}$  inside of the GLEM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. The left and right parts of the curved electric wave fronts are intersected at a branching point in topside. These branching points form a 2D subsurface which depends on the source location. In this Figure, the red S denotes the source which is located in the bottom of

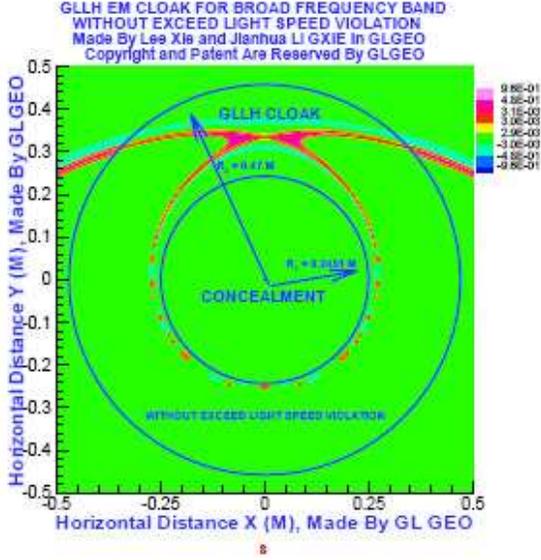


FIG. 20: (color online) At time step  $119dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. The red S in bottom is source.

the cloak. The front branching point is located in topside of the concealment.

## V. INVISIBLE PROPERTIES OF GLLH EM CLOAK

By using the GL EM Metro Carlo inversion method in [3], we solve the GL EM cloak inversion (11), (12), (13),(14) and (15), and obtain the GLLH EM invisible cloak . The invisible properties of the GLLH EM cloak are verified in this section.

### A. 1. The Invisibility of The GLEM Cloak

**Statement 1** The exterior EM field is not scattering interfered by the cloak

Because our GLLH EM exterior cloak inversion, by substituting the GLLH EM cloak in (2)-(5) and EM wave field by GL modeling [2] into (11) and (12), such that the integral equation (11) and (12) were held, we have identity (6) for any exterior source and frequency. therefore, any exterior EM wave field must not be scattering interfered by the cloak. By the 3D GL EM modeling [2] simulations, electric wave propagations in the figures 5-18 verify that there is no any reflection and no any scattering from the cloak to interfere the exterior EM field in free space.

By calculations, when  $R_i = 0.2491$  meter and  $R_o = 0.47$  meter, we can prove that on the outer boundary of

the cloak  $r = R_o$ ,  $\bar{\epsilon}$  and  $\bar{\mu}$  in (2)-(5) satisfy

$$\begin{aligned} \epsilon_r = \epsilon_\theta = \epsilon_\phi = 1, \quad r \in R_o, \\ \mu_r = \mu_\theta = \mu_\phi = 1, \quad r \in R_o. \end{aligned} \quad (17)$$

This is necessary condition for the invisibility of the cloak. From the GLLH EM cloak exterior inversion (11) and (12), we can prove that the condition (17) is necessary.

**Statement 2** Any exterior EM wave field can not penetrate into the concealment.

From the GLLH EM inner cloak inversion (13) and (14), and EM integral equation (8) and (9), the statement 2 is proved. Also, because on the inner boundary  $r = R_i$ , the refractive index  $\sqrt{\epsilon_r \mu_\theta}$  or  $\sqrt{\epsilon_\theta \mu_r}$  in (4)-(5) is infinite, from the convergence of integral (11)-(14), the EM wave field must be vanished on the inner boundary  $r = R_i$ . We obtain the condition

$$\begin{aligned} E_\theta = E_\phi = 0, \quad r \in R_i \\ H_\theta = H_\phi = 0, \quad r \in R_i \end{aligned} \quad (18)$$

Inversely, the conditions (17) and (18) are sufficient condition for the invisibility of the GLLH EM cloak in .

### B. Without Exceed Light Speed Violation

**Statement 3** The refractive index in the GLLH EM cloak configuration in this paper is large than one, the group speed in the cloak is finite and less than one.

From (4), the  $\epsilon_r$  and  $\mu_r$  are finite positive and monotone increase function on variable  $r$  in the  $R_i \leq r \leq R_o$ . moreover, their minimum on the inner boundary of the cloak are

$$\epsilon_r(R_i) = \mu_r(R_i) = \frac{2R_o^2(R_o - R_i)}{R_i^2(1 + 4R_o + 2R_o^2)} > 0, \quad (19)$$

their maximum value on the outer boundary  $R_o = 0.47$  of the cloak are one.

From (5), the  $\epsilon_\theta$  and  $\mu_\theta$  are monotone decrease function on variable  $r$  in the  $R_i \leq r \leq R_o$ , moreover, their maximum value on the inner boundary of the cloak are infinite, their minimum value on the outer boundary,  $R_o = 0.47$  meter of the cloak are one

The refractive index of the GLLH EM cloak in (2)-(5) is,

$$n(r) = \sqrt{\epsilon_r \mu_\theta} = \sqrt{\epsilon_\theta \mu_r} \geq 1. \quad (20)$$

The group speed is finite and  $V_g \leq 1$ . Therefore, in the annular configuration  $R_i \leq r \leq R_o$ , with  $R_i = 0.24$  and  $R_o = 0.47$  The GLLH EM invisible cloak material in (2)-(5) has no violation of exceeding light speed. The novel electric intensity wave propagations, which are shown in Figure 5 -18, in the GLLH EM cloak in (2)-(4) verify that the GLLH EM cloak is invisible cloak without exceed light speed violation.

C. Wave Front Branching

The EM wave propagation in the GLLH EM cloak is completely novel that are displaying in Figure 1 and Figure 2 and in Figures from 10 to 19. In particular, in Figure 14, EM wave front is curved around the concealment and its upside and downside two parts intersect at a moveable branching point, which form a 2D subsurface. The Figure 15 shows that the two wave fronts are separately propagation from the branching point. The first wave front is forward propagating outgoing. The second wave front is attracting propagating and approaching to the inner boundary  $r = R_i$ . The speed of two wave front propagation are shower then the light speed. The second wave front attracting propagating is very shower. Its amplitude is rapidly decay to zero. Its propagation speed is going to zero. The front branching points form a moveable 2D subsurface which depend on the source location. The front branching point and source point are separately located on two sides of the concealment room. In Figure 14 and Figure 15, the source is located in the right side of the cloak which is denoted by red S, the front branching point is located at the left side of the concealment. In the Figure 19, the source, which is denoted by red S, is located in left of the cloak, the front branching point is located in the right side of the concealment. In the Figure 20, the source, which is denoted by red S, is located in bottom of the cloak, the front branching point is located in the top side of the concealment. These novel EM wave propagations in our GLLH EM cloak and wave front branching phenomena only display in our paper. They never appear in other cloaks and never appear in other materials. The GLLH EM cloak , novel wave propagations and wave front branching are patent and copyright by authors in GL Geophysical Laboratory.

VI. GLLH EM DOUBLE LAYER CLOAK

We propose new GLLH EM double layer cloak in this section.

**Statement 4** When the concealment room of the single layer cloak is fill in the basic EM materials  $\epsilon_0$  and  $\mu_0$  or normal material with refractive index  $n \geq 1$ , and the cloak materials are finite, then there is no EM field can be excited by source inside of the concealment.

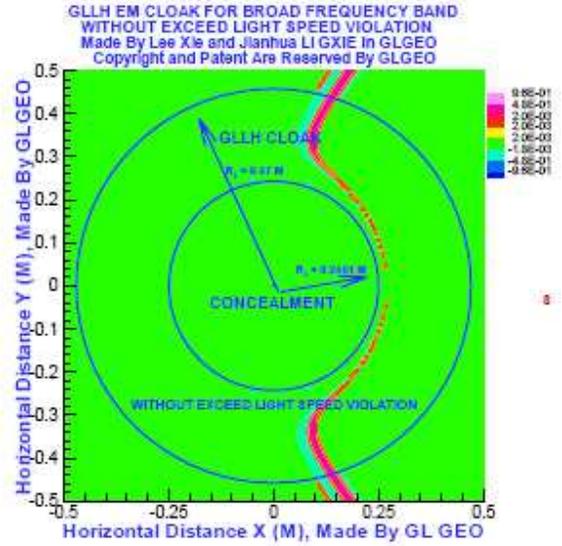


FIG. 21: (color online) At time step  $84dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed. The red S in right is source.

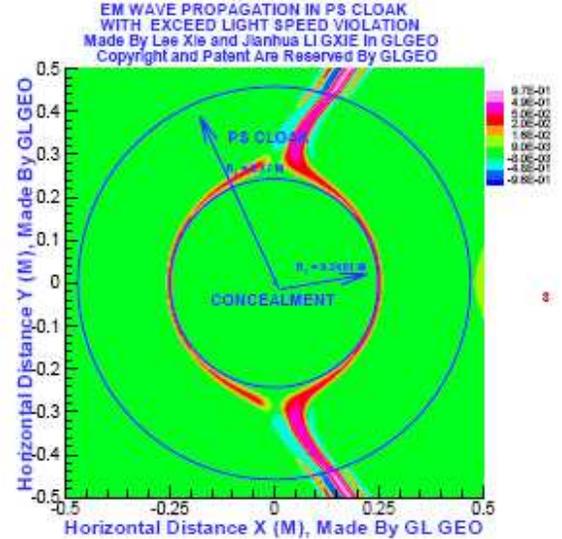


FIG. 22: (color online) At time step  $84dt$ , front of *Electric wave*,  $E_{xx}$  inside of the PS cloak  $R_1 \leq r \leq R_2$  propagation is split two phase around the concealment, the front phase exceed light speed, behind phase slower than light speed. The red S in right is source.

We have proved the statement in [10]and [15] . The *statement 4* theoretacly confirms that our double layer cloth phenomena in 2001 in [1] is a credible physical discovering. We propose GL double layer cloak in [9][11],[12],[13], and [16] and new GLLH EM double cloak here without exceed light speed violation. The inner layer cloak of The GLLH EM double layer is same as in (1) of [11] or (1) of [9],[12],[13], and [16], The GLLH EM cloak

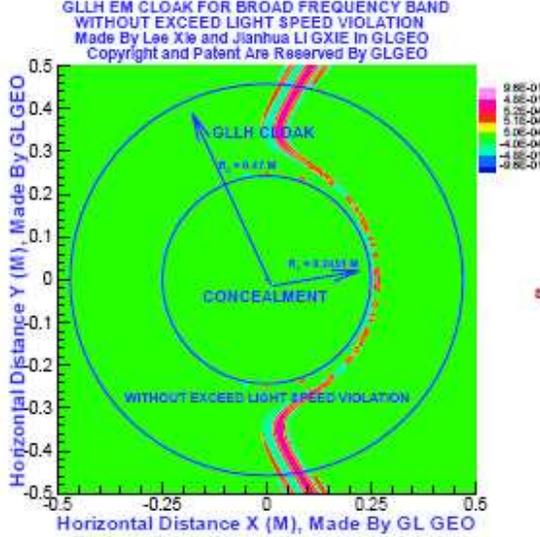


FIG. 23: (color online) At time step  $90dt$ , front of *Electric wave*,  $E_{xx}$  inside of the GLLH EM cloak  $R_1 \leq r \leq R_2$  propagates slower than light speed

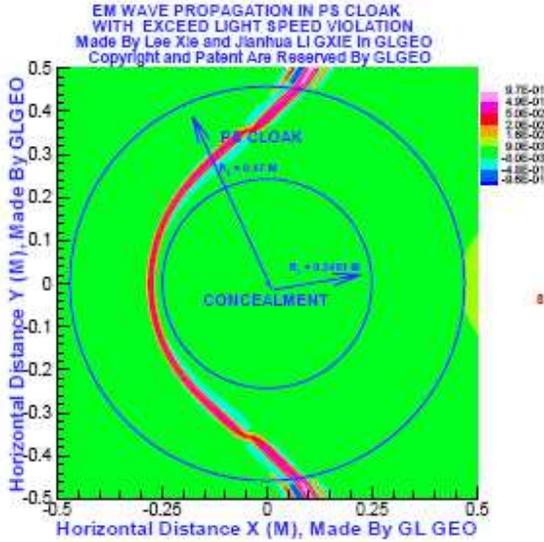


FIG. 24: (color online) At time step  $90dt$ , front of *Electric wave*,  $E_{xx}$  inside of the PS cloak  $R_1 \leq r \leq R_2$  propagation is much more exceed light speed. The red S in right is source.

is used as GL outer layer cloak of new GLLH EM double layer cloak. In the GL inner cloak, the EM wave can be excited by source inside of the concealment, when the EM wave propagates enter the GL inner layer cloak, it is rapidly decay, very slower than light speed and forward going to zero at outer boundary of the inner cloak. There is no any reflection back to the concealment from the GL inner layer cloak. The EM environment of its concealment is normal. The GL outer layer cloak has

full invisible functions for broad frequency band, has refractive index large than one, and has no exceed light speed violation. The reciprocal law is satisfied. Therefore, our GLLH EM double layer cloak has advantages to overcome the three physical violations in Ps cloak or in other cloak. The Ps cloak is single cloak with finite dielectric and permeability materials. By the *statement4*, there is no EM wave can be excited by source inside concealment of the Ps cloak. So Ps cloak's third physical violation caused that its concealment is dark room.

### VII. EM WAVE FIELD DECAY TO ZERO INVERSE RADIAL

**Statement 5**, When source is located outside of the GLLH invisible cloak in (2)-(5), the observer  $r$  is located inside of the cloak and is going to the inner boundary  $r = R_i$ , the EM wave field delay to zero inverse radial. We have

$$\begin{aligned} E_r(r, \theta, \phi) = E_\theta(r, \theta, \phi) = E_\phi = 0, \text{ when } r = R_i, \\ H_r(r, \theta, \phi) = H_\theta(r, \theta, \phi) = H_\phi = 0, \text{ when } r = R_i, \end{aligned} \quad (21)$$

where  $r = R_i$  is the inner boundary of the GLLH cloak, on the inner boundary  $r = R_i$ , the radial electric wave field  $E_r(r, \theta, \phi) = 0$  and the radial magnetic wave field  $H_r(r, \theta, \phi) = 0$  are distinctive property of the GLLH invisible cloak. On the inner boundary of other transformed cloak, the radial wave EM field are not zero.

### VIII. HISTORY AND DISCUSSION

We discovered the GILD double layer cloth phenomena [1] to prevent detection from the exterior EM wave in 2001. The double layer cloth was appeared in residual magnetic field  $H_y$  in the GILD EM modeling and inversion [1]. Many times repeat GILD simulations show that the GILD double layer cloth is a mathematical physical phenomenon. We develop complete new GL EM modeling [2] and GL EM inversion [3][courant] in 2003 to investigate the strange phenomena. We did find mirage [5] and double layer cloaks [9-13]. Our GLLH EM cloaks in this paper and in previous papers are based on the GL EM modeling and inversion and number theory. We deeply investigate the relationship between the field and material interactive scattering and non scattering and propose the GLLH EM cloak inversion. Our GL methods do not need artificial boundary condition to truncate the infinite domain. The GL method does not need to solve big matrix equation. The GL method combines the analytical method and numerical method. It can perform analytical and numerical or mixed field simulation and material generation. Front branching is complete novel propagation. Attracting front has benefit for detect the exterior field information in invisible cloaked concealment.

The GLLH EM cloak materials (4)-(5) is similar with the form  $N^\alpha / \log^\beta N$  in number theory [20][21][22] and

$h^\alpha \log^\beta(1/h)$  in superconvergent estimation [5][23]. That is consistent with our double layer cloth phenomena discovered in the residual field in [1]. By paper [20] and [21], GOLDBACH's conjecture proof will have a big step forwarding.

The electromagnetic physics is governed by Maxwell equation. The equation is main tool for physics that relative both of geometry and number theory. Our GL EM full wave field modeling and inversion are based on the Maxwell equation The form  $N^\alpha/\log^\beta N$  and  $h^\alpha \log^\beta(1/h)$  are important mathematical form in the number theory and numerical estimation A good luck thing is that we chose these form and their appreciate combination and GL EM inversion to discover the invisible cloak material in without exceed light speed. Other cloaks based on the coordinate geometry transformation have exceed light speed and infinite speed physical violations.

To compare with Ps cloak which is based on the coordinate transformation, we simulate EM full wave propagation through the Ps cloak material using our GL EM modeling with same source, frequency band and domain geometry as our GLLH EM cloak simulation. In Figure 21, at 84 time step, the EM wave propagates in the GLLH EM cloak and slower than the light speed. Its front inside the GLLH EM cloak is behind of the front in the free space. In Figure 22, at 84 time steps, the EM wave propagates through Ps cloak, its front is split to two phases, its front phase exceed the light speed. At 90 time steps, the Figure 24 shows that the speed of EM wave inside Ps cloak much more exceed the light speed. At 90 time step, the EM propagation in GLLH EM cloak is presented in Figure 23, it is obvious that the EM wave front inside GLLH EM cloak is behind of its front in the free space which shows that the speed of EM wave inside GLLH EM cloak is not exceeding light speed. We propose GLLH EM cloak without exceed light speed in ellipsoid, 2 polar geometry domain and other geometry domain. For overcoming the infinite speed in [14], Ulf Leonhardt et al. proposed a new cloak with finite speed based , on a Euclid and non Euclid joint transform[17]. The ULF cloak overcomes the "infinite speed" physical violation, even though its refractive index less than one in some subdomain, for example,  $n(\sigma, \sigma', \tau) = n(0.75\pi, \pi, \tau) < 1$ . Our GLLH EM cloak without exceed light speed in the 2 poplars geometry domain and in the ellipsoid domain will be published.

## IX. CONCLUSION

The GL EM modeling and inversion theoretical analysis and many GLLH EM modeling simulations verified

that the GLLH EM cloak has full invisible functions. The GLLH EM invisible cloaks have no *infinite speed* and *exceed light speed* two violations. The novel EM wave propagations have presented in Figure 1-2 and Figure 5-18. The EM wave front inside of the GLLH EM cloak has always been behind of the wave front in the free space. The front inside of the GLLH EM cloak completely does curve around the concealment and form curved Crescent-like in Figure 1 and 13. The two front teeth of the Crescent front intersect at the branching point in Figure 14, 19 and 20. The branching points form 2D subsurface. The wave front is split to two fronts at the front branching point. The outgoing front propagates forward outgoing and recovers to original wave front in free space. The attractive front propagates and to the shrinks to inner boundary of the cloak. Its amplitude and speed are rapidly decay to zero. Using GLLH EM cloak as outer layer, the GLLH EM double cloak has normal environment concealment. In its concealment, the EM wave field excited by the internal source propagates enter to the inner layer cloak, its amplitude and speed rapidly decay to zero before the outer boundary of the inner layer cloak. Therefore, the GLLH EM double invisible cloak overcomes the three physical violations in other cloaks. GLLH EM double invisible cloak can be practicable. GLLH EM cloak software can generate various scale full invisible cloaks without exceed light speed, and can generate GLLH EM double cloak for various scale that have no physical violation and have wide applications. GLLH EM cloak and GLLH EM cloak modeling and inversion and GLLH EM cloak software are patented and copyrighted by authors in GL Geophysical Laboratory

The GL modeling and inversion method is an effective physical simulation method. It has double abilities of the theoretical analysis and numerical simulations to study the cloak metamaterials and wide material and Field scattering in physical sciences.

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