Invisibility cloaking without superluminal propagation

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Abstract

Conventional cloaking based on Euclidean transformation optics requires that the speed of light should tend to infinity on the inner surface of the cloak. Non-Euclidean cloaking still needed media with superluminal propagation. Here we show by giving an example that this is no longer necessary.

1 Introduction

The development of transformation optics [1–10] and metamaterials [11–16] had lead to the theoretical prediction of invisible dipoles [17, 18] and invisibility cloaks [4, 5]. Since then the principles behind invisibility have been demonstrated in a number of experiments that feature cloaking in some reduced form. Electromagnetic cloaking was first realised for microwaves of one frequency and polarisation [19]. Numerous subsequent electromagnetic experiments achieved "carpet cloaking" [20–26]. A carpet cloak does not make things invisible, but makes them appear to be flat. Approximate cloaking has also been realised through tapered waveguides [27, 28]. Invisibility by plasmonic covering [17] was demonstrated as well [29]. Furthermore, non-electromagnetic forms of cloaking have been demonstrated for acoustic waves [30, 31].

However, the realisation of electromagnetic cloaking suffers from a practical and a fundamental problem [32–34]. The practical problem is that the material requirements for electromagnetic cloaking are very difficult to meet, whereas the fundamental problem is that perfect invisibility [5] requires that light should propagate in certain cloaking regions with a superluminal phase velocity that tends to infinity. In principle, this can be achieved by metamaterials, but only for discrete frequencies that correspond to the resonant frequencies of the cloaking material [6,34]. In contrast, acoustic cloaking [30,31] is much easier to achieve, since acoustic waves are not effected by relativistic causality and thus they are not restricted by their velocity in air.

Broadband cloaking based on non-Euclidean geometries has been proposed in [32,33] to avoid the requirement for infinite light velocities for electromagnetic cloaking. In this proposal the speed of light is finite in the entire cloaking region and, therefore, this cloak has the potential of working for a broad range of frequencies. However a fundamental problem still remains. Since space has to be expanded to make room for the invisible region within the cloak, the implementation of this device will still demand superluminal propagation (i.e. propagation with a velocity that exceeds the speed of light in vacuum). The same is true for "carpet cloaking" [20–26] where the velocity of light in the cloaking device must exceed the speed of light in the host material, i.e. vacuum if such devices were to find practical applications.

In this paper we give an example of a device that achieves complete electromagnetic cloaking—not just "carpet cloaking"—while all light velocities within the cloak are finite and less than the speed of light[1]. Through this example we demonstrate that invisibility cloaking is possible without superluminal propagation and anomalous material requirements.

2 Problem

Since transformation optics establishes a one-to-one correspondence between spatial geometries and dielectric media, invisibility can be visualised in terms of virtual geometries. There-

[1]The preprint [35] proposes a different method for cloaking without superluminal propagation.

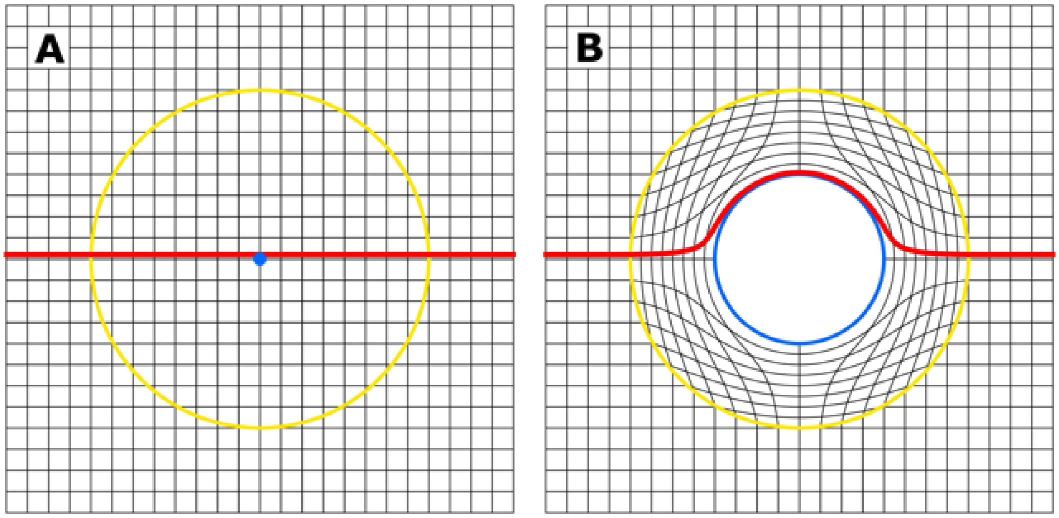

Figure 1: A Euclidean cloaking device expands a single point in virtual space (blue dot in A) into an extended region in physical space (blue circle in B) through the curved transformation of the virtual coordinate grid. A light ray that is smoothly guided around the invisible region in physical space (B) appears to have passed through empty space (A), making the region within the blue circle (B) invisible. However, along the blue circle the phase velocity of light goes to infinity.

fore, we explain the main ideas of this paper in terms of pictures. The complete calculations behind these pictures can be found in the Appendix.

First, let us contrast our method to conventional cloaking based on coordinate transformation. Figure 1 explains the conventional cloaking method described in [5]. A single point in virtual space – also called electromagnetic or optical space – is expanded into a finite region in physical space. The expansion is effected by the curved transformation of the straight virtual coordinate lines. Consequently, a light ray that traces out a curved trajectory in physical space (Fig. 1B) appears to propagate along a straight line in virtual space (red line in Fig. 1A). This creates the illusion that light has propagated through empty space, making the central region in physical space (region bounded by the blue circle in Fig. 1B) invisible.

However, there is a fundamental problem with this device. Light crosses the central point in virtual space (blue dot in Fig. 1A) in an infinitely short time. This point turns into an extended region in physical space, which still has to be traversed in an infinitely short instant. Therefore, the phase velocity of light will tend to infinity along the outer boundary of the invisible region in physical space. Infinite light velocities can be achieved by metamaterials, but only for discrete resonant frequencies of the material [32, 34], fundamentally restricting the applicability of this cloak.

In contrast, the broadband non-Euclidean invisibility device proposed in [32] achieves full

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